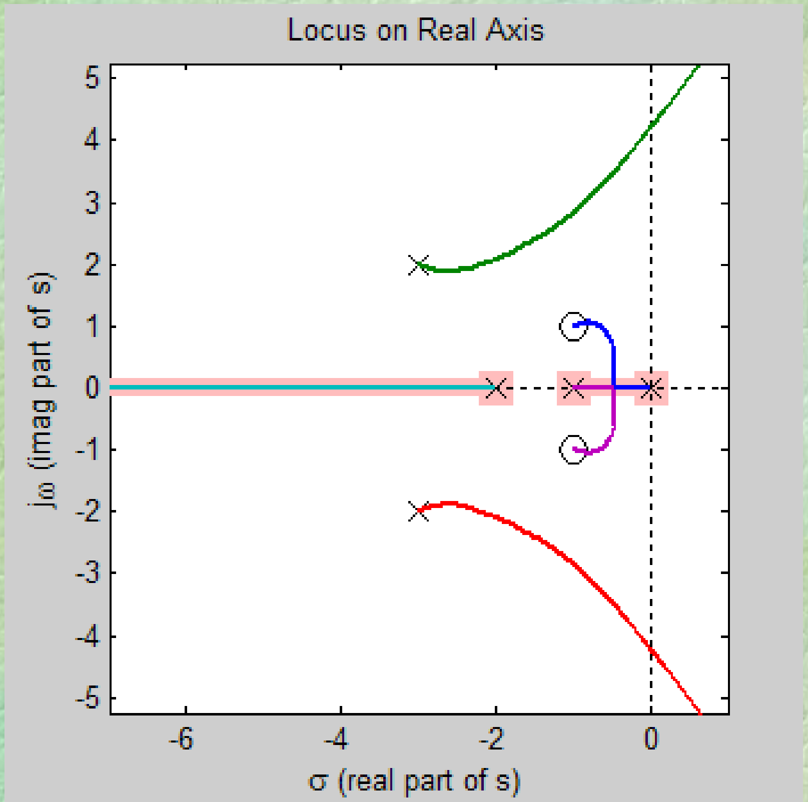
Root Locus

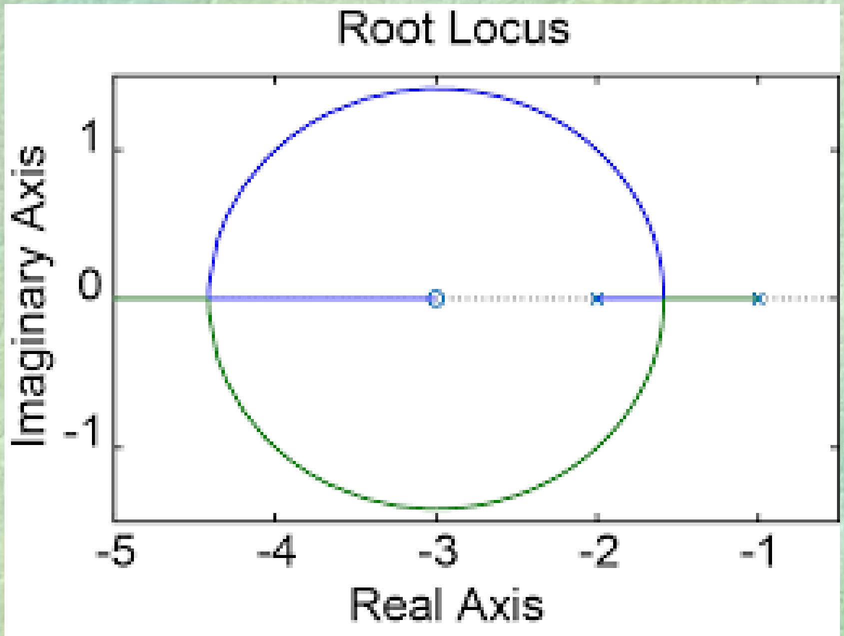
- It is introduced by W.R Evans in 1948 for the analysis of control systems.

- It is a powerful tool for adjusting the location of closed loop poles to achieve the desired system performance by varying one or more system parameters.

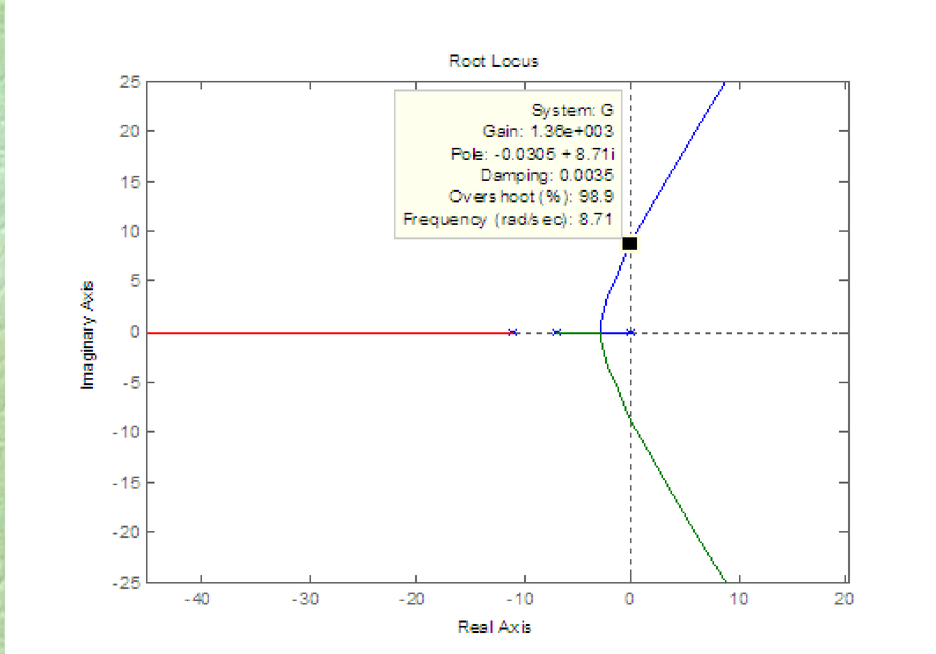
- It is a powerful method of analysis and design for stability and transient response.

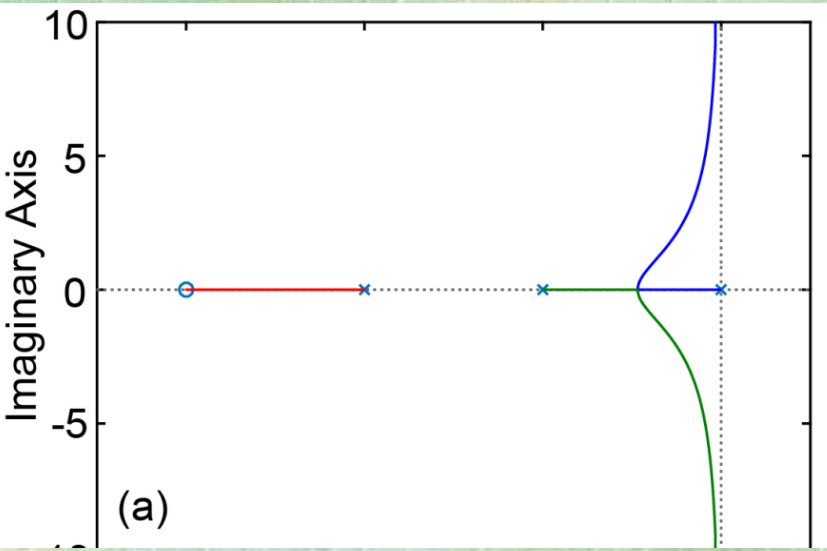
Typical Sketches of Root Locus Plots





TYPICAL SKETCHES OF ROOT LOCUS PLOTS ….





Procedure for constructing Root Locus

Step 1:-

- **Locate the Poles and Zeros of G(s).H(s) on the S-plane.**
- **The Root locus branch start from open loop poles and terminate at Zeros.**

n – number of poles, m – number of zeros

Step 2:-

- **Determine the Root locus on real axis.ie, Take a test point on real axis.**

Procedure for constructing Root Locus….

Step 3:-

➢**Determine the asymptotes of Root locus branches and meeting point of asymptotes with real axis and Centroid.**

i)Angle of Asymptote = [±180°(2q+1)]/(n-m),
q=0,1,2,3…(n-m)

ii) Centroid= [sum of poles-sum of zeros]/[n-m]

Procedure for constructing Root Locus….

Step 4:-

- **Find the break-away and break-in points.**

  i) If there is a Root locus on real axis between 2 poles then there exist a break-away point

  ii) If there is a Root locus on real axis between 2 zeros then there exist a break-in point

  iii) If there is a Root locus on real axis between poles and zeros then there may be or may not be break-away or break –in point

Procedure for constructing Root Locus….

Step 5:-

- **If there is <u>complex pole</u> then determine the angle of departure from the complex pole.**

Angle of Departure = [180°-(sum of angles of vector to the complex pole A from other poles) + (sum of angles of vector to the complex pole A from other zeros)]

Procedure for constructing Root Locus….

If there is complex zero then determine the angle of arrival at the complex zero.

Angle of Departure = [180°-(sum of angles of vector to the complex zero A from all other zeros) + (sum of angles of vector to the complex zero A from other poles)]

Procedure for constructing Root Locus….

Step 6:-

- **Find the points where the Root loci may cross the imaginary axis.**

Step 7:-

- **Take a series of test point in the broad neighborhood of the origin of the S-plane and adjust the test point to satisfy angle criterion.**

Procedure for constructing Root Locus….

- **Sketch the Root locus by joining the test point by smooth curve.**

- **If the total no. of poles and zeros on real axis to the right of test point is**

  i)Odd no. - Test point lies on real axis
  ii) Even no. - Test point does not lie on real axis

Procedure for constructing Root Locus….

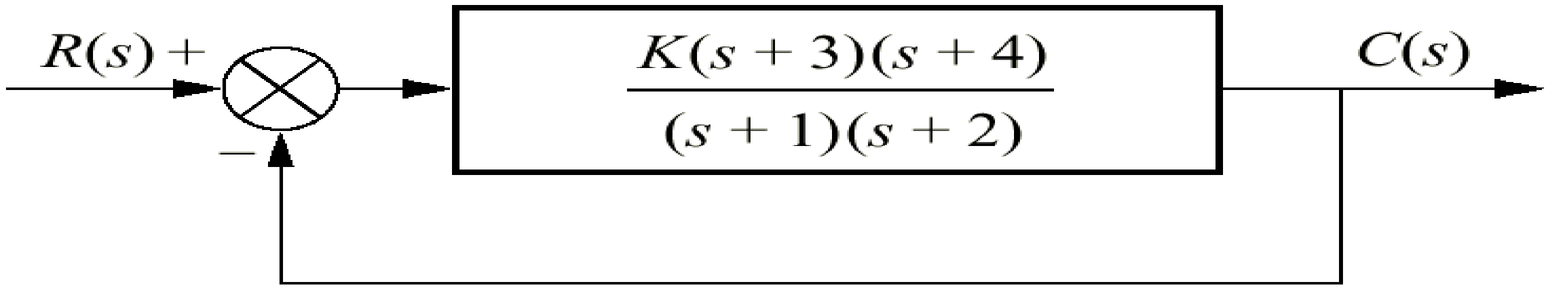
- **Step 8:-**

The value of gain K at any point on the locus can be determined from Magnitude condition.

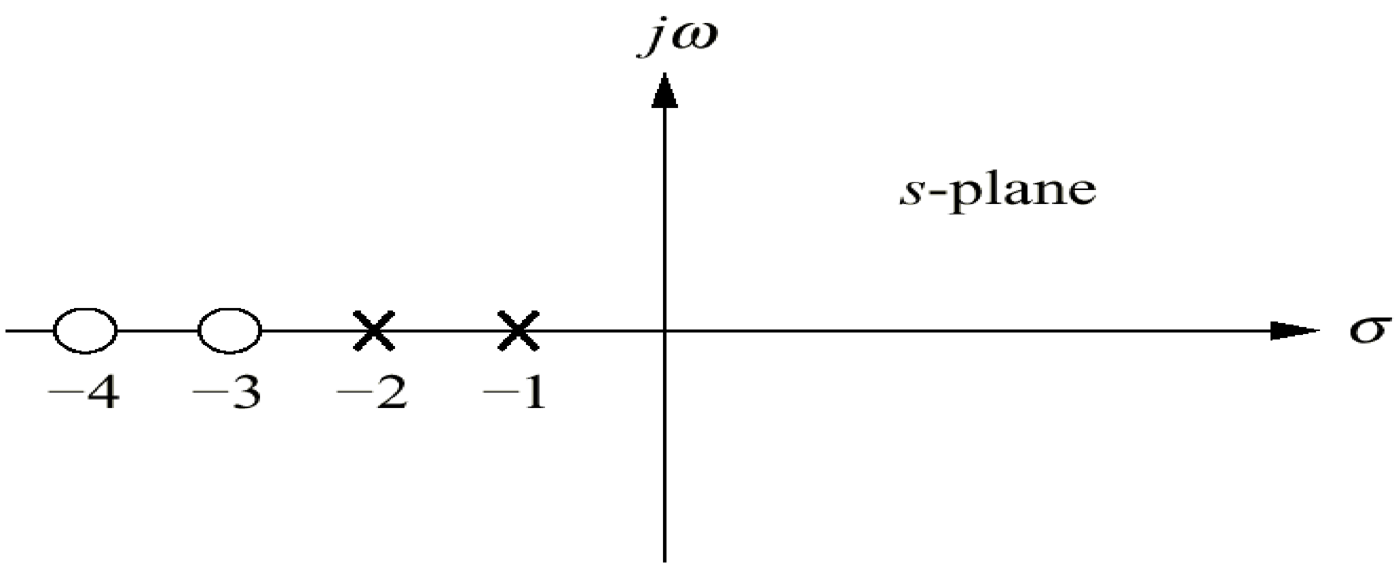
The magnitude condition is given by

Gain K at a point (S=Sa) = [Product of length of vectors from poles to the point S=Sa] / [Product of length of vectors from finite zeros to the point S=Sa]

Location of Poles and Zeros

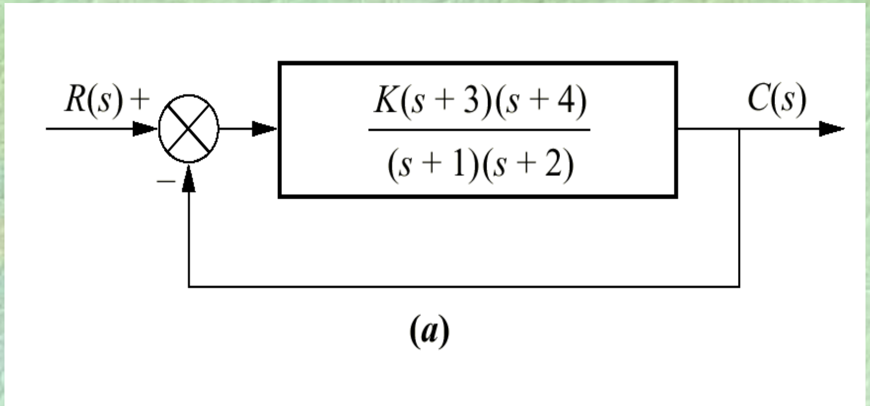


(a)

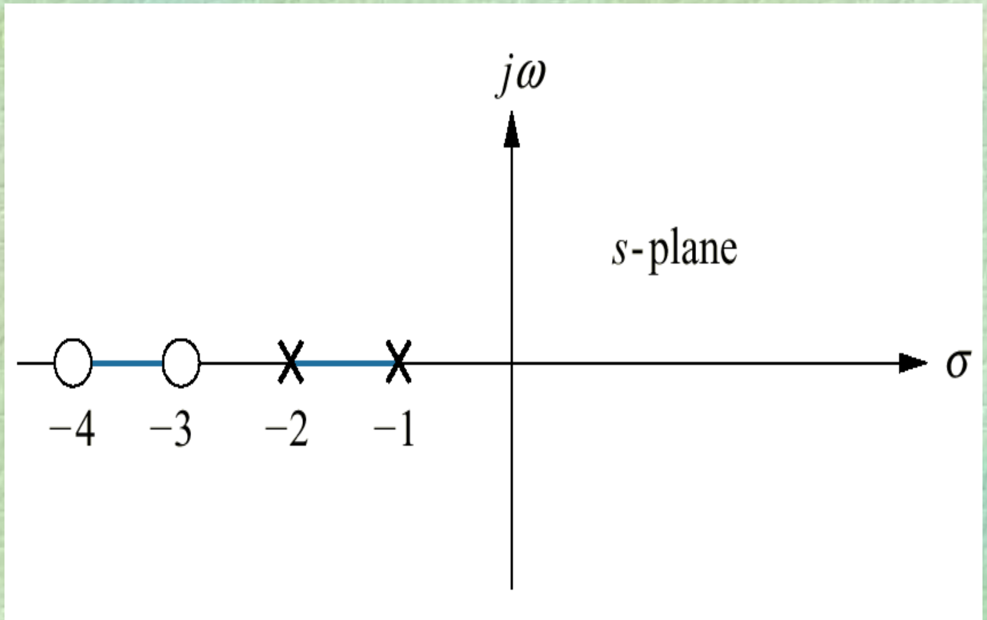


(b)

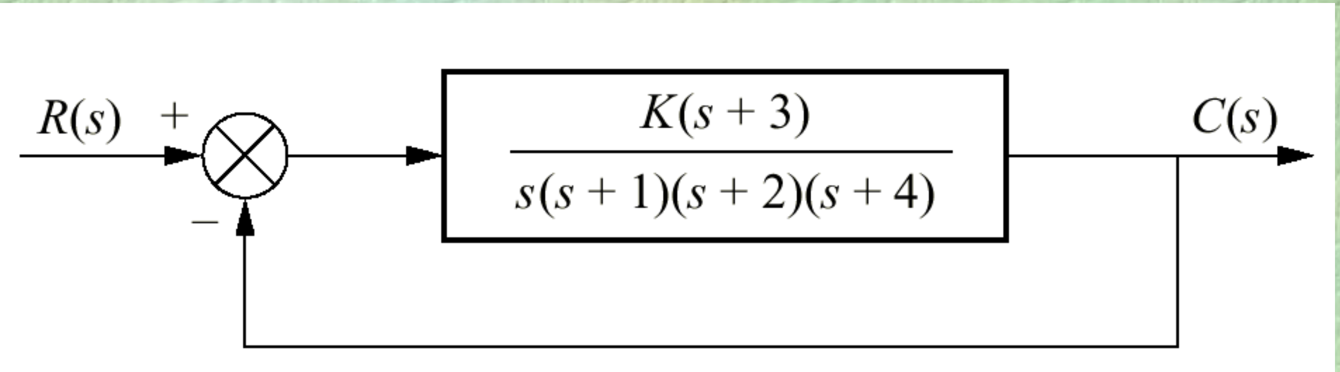
ROOT LOCUS ON REAL AXIS



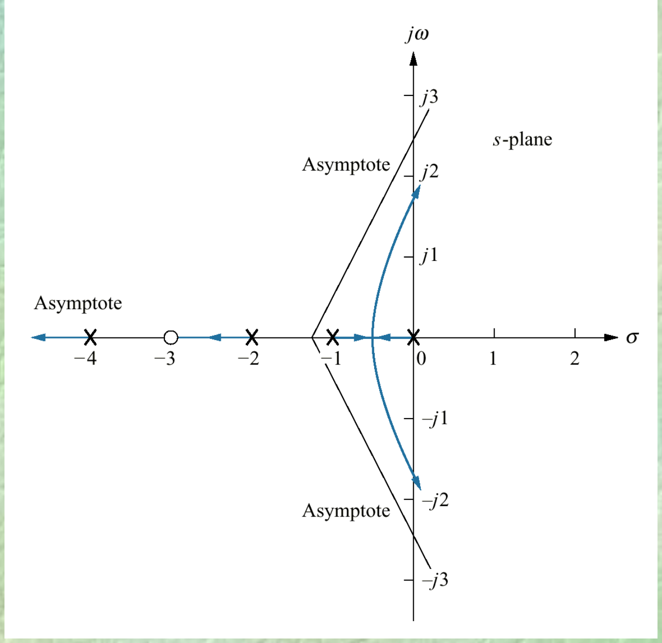
The root locus exists between (1- , -2) (-3,-4)



ASYMPTOTES AND CENTROID



$$\sigma_a = \frac{(-1-2-4)-(-3)}{4-1} = -\frac{4}{3}$$

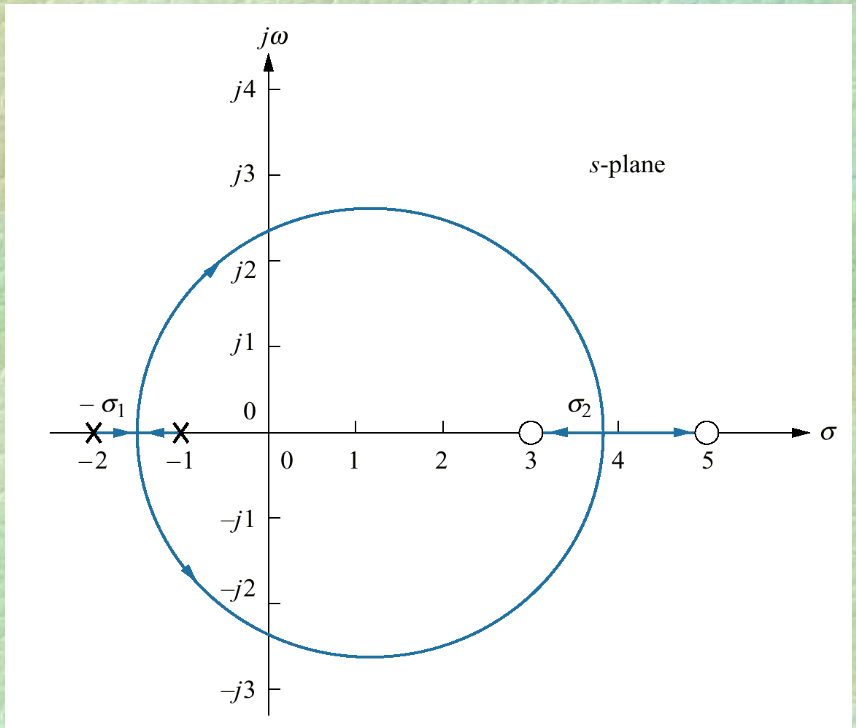


BREAK– AWAY POINTS

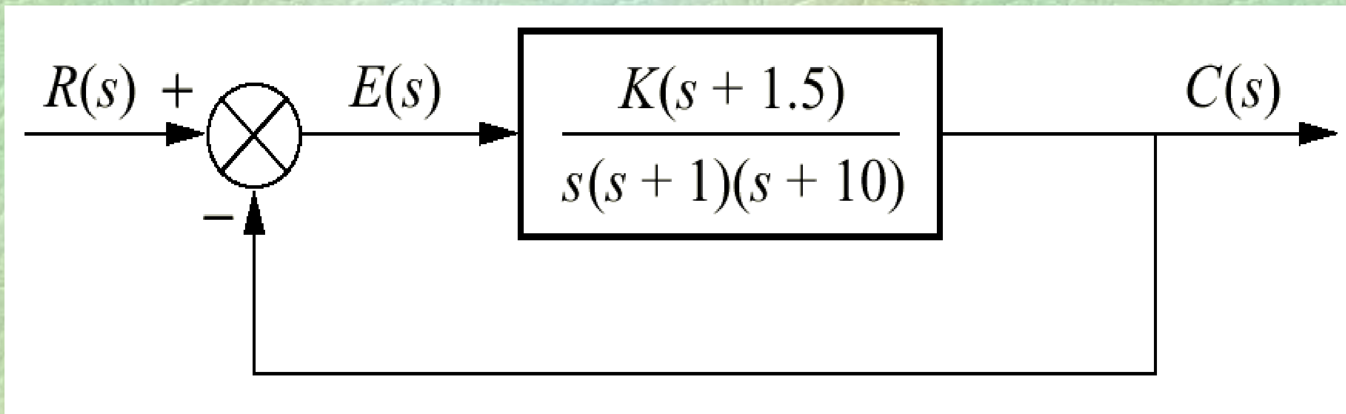
$$KG(s)H(s) = \frac{K(s-3)(s-5)}{(s+1)(s+2)} = \frac{K(s^2-8s+15)}{(s^2+3s+2)}$$

Solving for K, and then differentiating, We get

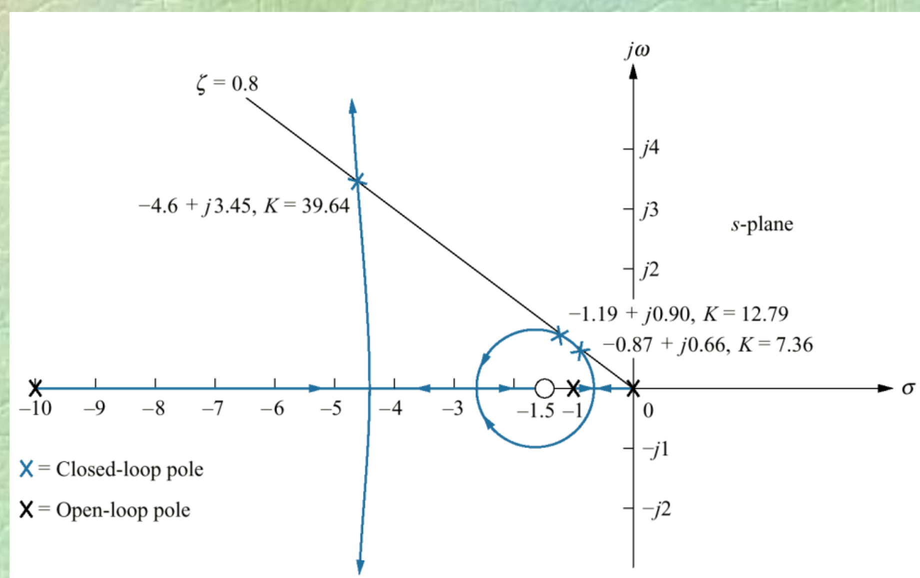
S= = -1.45 and 3.82



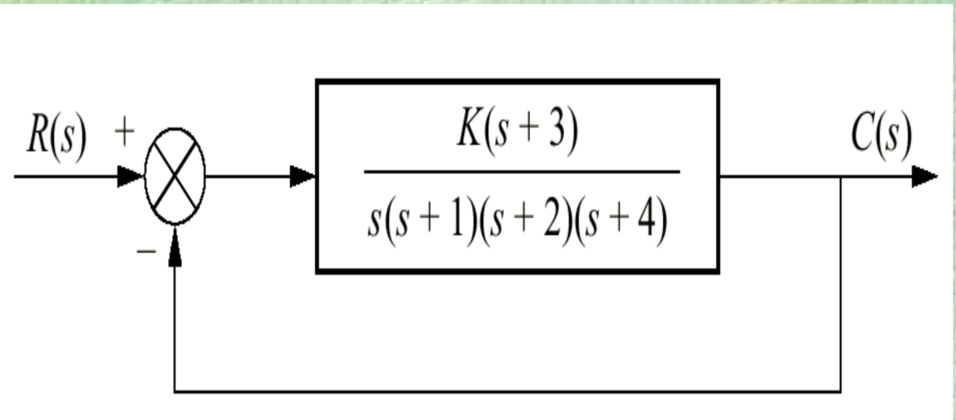
BREAK – IN POINTS

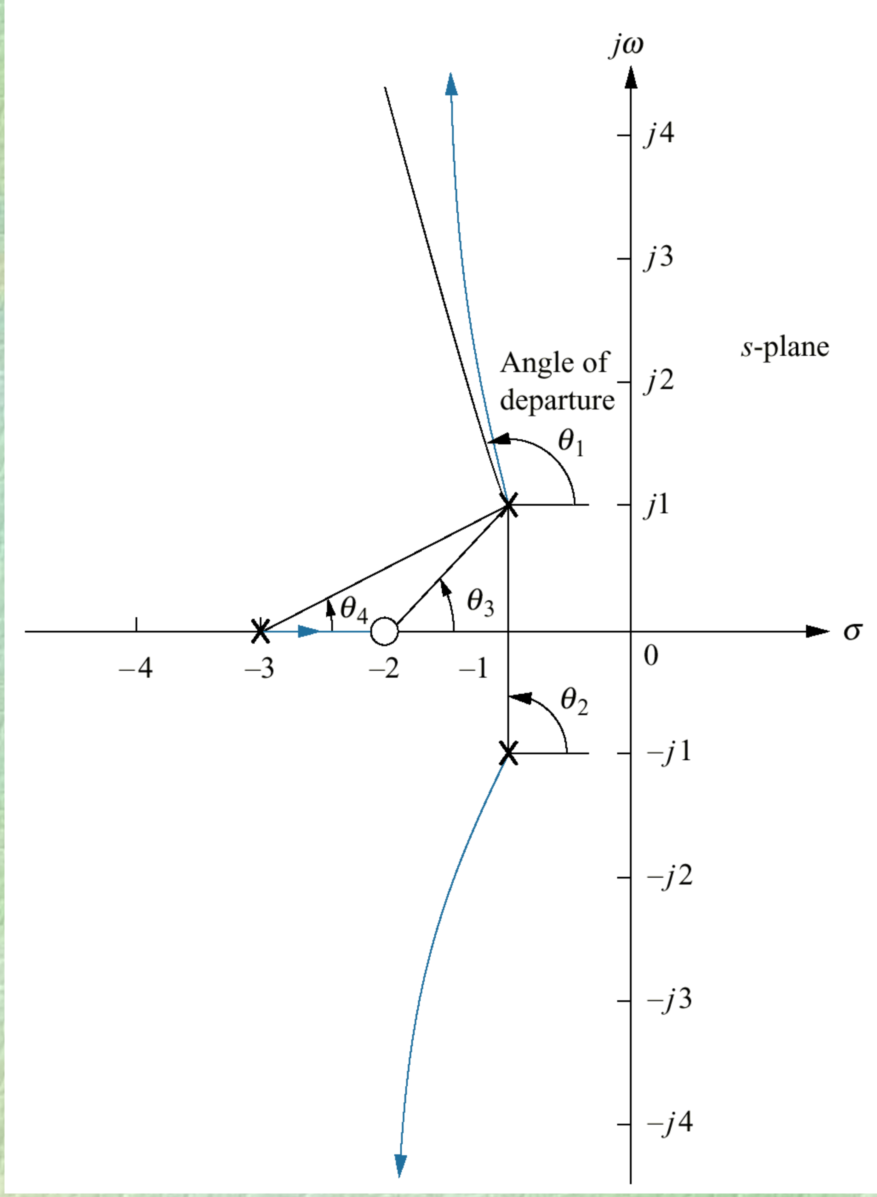


Break-in point is found at -2.8 with k = 27.91



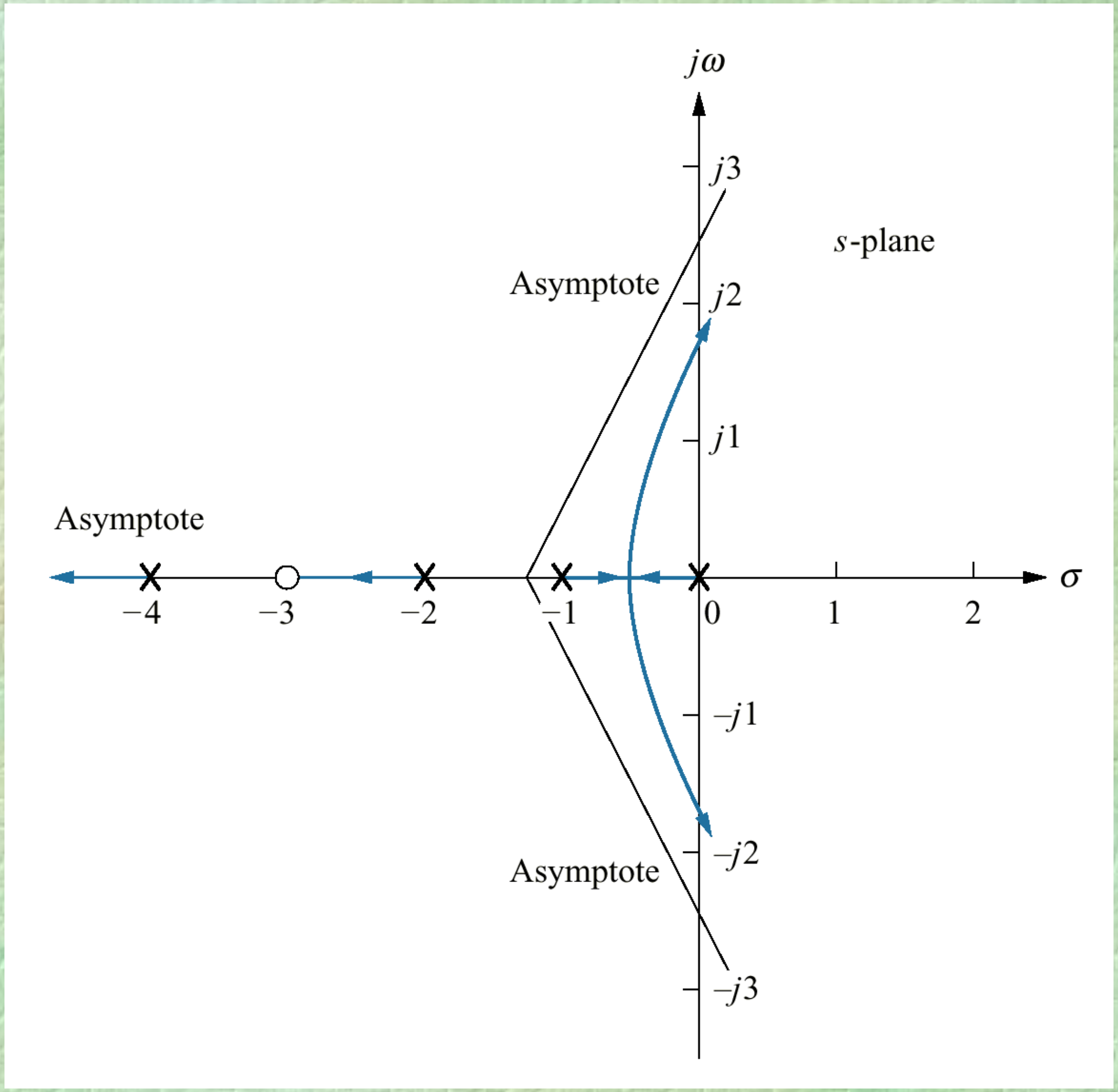
ANGLE OF DEPARTURE and ARRIVAL



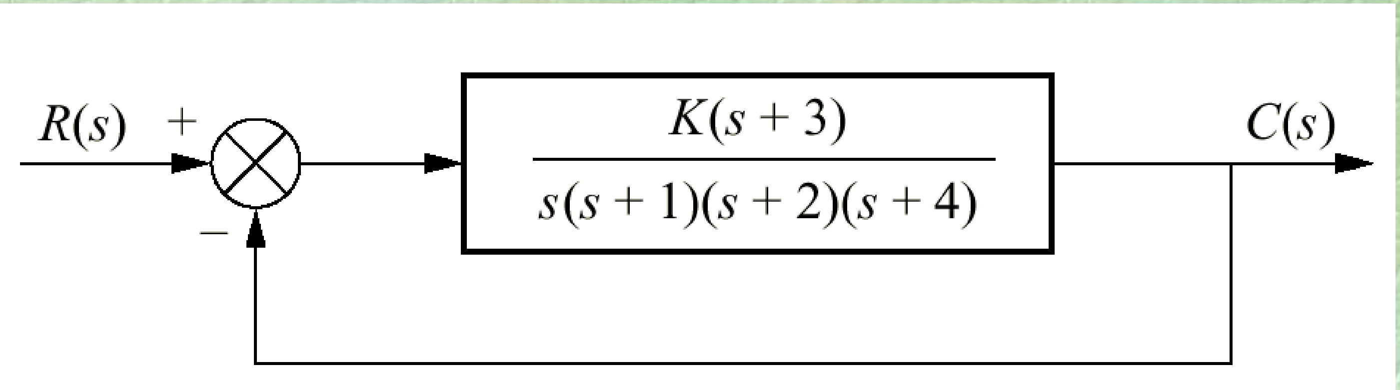


$$\theta_1 = -251.6^\circ = 108.4^\circ$$

ANGLE OF ARRIVAL



CALCULATION OF GAIN VALUE K



$$T(s) = \frac{K(s+3)}{s^4 + 7s^3 + 14s^2 + (8+K)s + 3K}$$

-K^2-65K+720 = 0 , and
solving for K= 9.65

CALCULATION OF GAIN VALUE K…

$$(90-K)s^2 + 21K = 80.35s^2 + 202.7 = 0$$

s^4	1	14	$3K$
s^3	7	$8+K$	
s^2	$90-K$	$21K$	
s^1	$\dfrac{-K^2-65K+720}{90-K}$		
s^0	$21K$		

S is found to be ±j1.59. Thus the root locus crosses the imaginary axis at ±j1.59 at a gain of 9.65. We conclude the system is stable for

$$0 \le K < 9.65$$